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SQUARE SUM PRIME LABELING OF SOME TREE GRAPHS

Sunoj B S,
Department of Mathematics, Government Polytechnic College, Attingal, Kerala, India
Email: spalazhi@yahoo.com

Mathew Varkey T K
Department of Mathematics, T K M College of Engineering, Kollam, Kerala, India
Email: mathewvarkeytk@gmail.com

Abstract

Square sum prime labeling of a graph is the labeling of the vertices with {0,1,2-------,p-1} and the edges with sum of the squares of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits square sum prime labeling. Here we identify some tree graphs for square sum prime labeling.

Key Words: Graph labeling, square sum, greatest common incidence number, prime labeling.

1.0 Introduction

All graphs in this paper are trees. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. The square sum labeling was defined by V Ajitha, S Arumugan and K A Germina in [5]. In this paper we introduced square sum prime labeling using the concept greatest common incidence number of a vertex. We proved that some tree graphs admit square sum prime labeling.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

2.0 Main Results

Definition 2.1 Let G = (V, E) be a graph with p vertices and q edges. Define a bijection
f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f_{sqsp}^{*} : E(G) → set of natural numbers N by $f_{sqsp}^{*}(uv) = \{f(u)\}^2 + \{f(v)\}^2$. The induced function f_{sqsp}^{*} is said to be a sum square prime labeling, if the ***gcin*** of each vertex of degree at least 2 is 1.

Definition 2.2 A graph which admits square sum prime labeling is called a square sum prime graph.

Theorem 2.1 Comb graph $P_n \odot K_1$ admits square sum prime labeling.

Proof: Let G = $P_n \odot K_1$ and let v_1,v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| =2n-1

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\ v_{i+1})$ = $2i^2 - 2i + 1$, i = 1,2,------------,n+1

$f^*_{sqsp}(v_{i+2}\ v_{n+i+2})$ = $(n+i+1)^2 + (i+1)^2$, i = 1,2,------------,n-2

Clearly f^*_{sqsp} is an injection.

gcin of (v_{i+1}) = gcd of $\{f^*_{sqsp}(v_i\ v_{i+1}),\ f^*_{sqsp}(v_{i+1}\ v_{i+2})\}$

= gcd of $\{2i^2+2i+1, 2i^2-2i+1\}$

= gcd of $\{4i, 2i^2-2i+1\}$,

= gcd of $\{i, 2i^2-2i+1\}$ = 1, i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_n \odot K_1$, admits square sum prime labeling. ■

Theorem 2.2 Star graph $K_{1,n}$ admits square sum prime labeling.

Proof: Let G = $K_{1,n}$ and let u,v_1,v_2,----------------,v_n are the vertices of G

Here |V(G)| = n+1 and |E(G)| =n

Define a function f : V → {0,1,2,3,----------------,n } by

$f(v_i) = i$, i = 1,2,------,n

f(u) = 0

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(u\ v_i)$ = i^2 , i = 1,2,------------,n

Clearly f^*_{sqsp} is an injection.

gcin of (u) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $K_{1,n}$, admits square sum prime labeling. ■

Theorem 2.3 Bistar B(m,n), admits square sum prime labeling.

Proof: Let G = B(m,n) and let a,b,v_1,v_2,--------------,v_m,u_1,u_2,---------------,u_n are the vertices of G

Here |V(G)| = m+n+2 and |E(G)| = m+n+1

Define a function f : V → {0,1,2,3,----------------,m+n+1 } by

$f(v_i) = i+1$, i = 1,2,------,m

$f(u_i) = m+i+1$, i = 1,2,------,n

f(a) = 0 , f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(a\ v_i)$ = $(i+1)^2$, i = 1,2,------------,m

$f^*_{sqsp}(a\ u_i)$ = $(m+i+1)^2 + 1$, i = 1,2,------------,n

$f^*_{sqsp}(a\ b)$ = 1.

Clearly f^*_{sqsp} is an injection.

gcin of (a) = 1.

gcin of (b) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence B(m,n), admits square sum prime labeling. ■

Theorem 2.4 Coconut tree graph CT(m,n), admits square sum prime labeling.

Proof: Let G = CT(m,n) and let v_1,v_2,----------------,v_{m+n} are the vertices of G

Here |V(G)| = m+n and |E(G)| =m+n-1

Define a function f : V → {0,1,2,3,----------------,m+n-1 } by

$f(v_i) = i-1$, i = 1,2,------,m+n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{sqsp} is defined as follows

$f^*_{sqsp}(v_i\ v_{i+1})$ = $2i^2 - 2i + 1$, i = 1,2,------------,m-1

$f^*_{sqsp}(v_m\ v_{m+i})$ = $(m+i-1)^2 + (m-1)^2$, i = 1,2,------------,n

Clearly f^*_{sqsp} is an injection.

gcin of (v_{i+1}) = 1, i = 1,2,------------,m-1.

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence CT(m,n), admits square sum prime labeling. ■

Theorem 2.5 Centipede graph C(2,n) admits square sum prime labeling.
Proof: Let G = C(2,n) and let v_1,v_2,----------------,v_{3n} are the vertices of G
Here |V(G)| = 3n and |E(G)| =3n-1
Define a function f : V → {0,1,2,3,----------------,3n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,3n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{sqsp}^* is defined as follows

$f_{sqsp}^*(v_{3i-2}\ v_{3i-1})$ = $18i^2 - 30i + 13$, i = 1,2,------------,n
$f_{sqsp}^*(v_{3i-1}\ v_{3i})$ = $18i^2 - 18i + 5$, i = 1,2,------------,n
$f_{sqsp}^*(v_{3i-1}\ v_{3i+2})$ = $18i^2 - 6i + 5$, i = 1,2,------------,n-1

Clearly f_{sqsp}^* is an injection.
gcin of (v_{3i-1}) = gcd of $\{f_{sqsp}^*(v_{3i-2}\ v_{3i-1}), f_{sqsp}^*(v_{3i-1}\ v_{3i})\}$
= gcd of { $18i^2$-30i+13, $18i^2$-18i+5}
= 1, i = 1,2,------------,n
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence C(2,n) , admits square sum prime labeling. ■

Theorem 2.6 Twig Graph $T_w(n)$, admits square sum prime labeling.
Proof: Let G = $T_w(n)$ and let v_1,v_2,----------------,v_{3n-4} are the vertices of G
Here |V(G)| = 3n-4 and |E(G)| =3n-5
Define a function f : V → {0,1,2,3,----------------,3n-5 } by
$f(v_i)$ = i-1 , i = 1,2,------,3n-4
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{sqsp}^* is defined as follows

$f_{sqsp}^*(v_i\ v_{i+1})$ = $2i^2 - 2i + 1$, i = 1,2,------------,n-1
$f_{sqsp}^*(v_{n-i}\ v_{n+i})$ = $(n-i-1)^2 + (n+i-1)^2$, i = 1,2,------------,n-2
$f_{sqsp}^*(v_{n-i}\ v_{2n-2+i})$ = $(n-i-1)^2 + (2n+i-3)^2$, i = 1,2,------------,n-2

Clearly f_{sqsp}^* is an injection.
gcin of (v_{i+1}) = 1, i = 1,2,------------,n-2
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $T_w(n)$, admits square sum prime labeling. ■

Theorem 2.7 H-graph of path P_n admits square sum prime labeling.
Proof: Let G = $H(P_n)$ and let v_1,v_2,----------------,v_{2n} are the vertices of G
Here |V(G)| = 2n and |E(G)| =2n-1
Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f_{sqsp}^* is defined as follows

$f_{sqsp}^*(v_i\ v_{i+1})$ = $2i^2 - 2i + 1$, i = 1,2,------------,n-1
$f_{sqsp}^*(v_{n+i}\ v_{n+i+1})$ = $(n+i-1)^2 + (n+i)^2$, i = 1,2,------------,n-1

Case(i) n is odd

$f_{sqsp}^*(v_{(\frac{n+1}{2})}\ v_{(\frac{3n+1}{2})}) = \frac{5n^2-4n+1}{2}$

Case(ii) n is even

$f_{sqsp}^*(v_{(\frac{n+2}{2})}\ v_{(\frac{3n}{2})}) = \frac{5n^2-6n+2}{4}$

Clearly f_{sqsp}^* is an injection.
gcin of (v_{i+1}) = 1, i = 1,2,------------,n-2
gcin of (v_{n+i+1}) = 1, i = 1,2,------------,n-2
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $H(P_n)$, admits square sum prime labeling. ■

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