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STOCHASTIC ANALYSIS OF A TWO DISSIMILAR UNIT SYSTEM WITH THE PROVISO OF REST FOR ORDINARY UNIT

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Abstract: This paper deals with the stochastic analysis of a two dissimilar unit system with the proviso of rest to ordinary unit. Using regenerative point technique with Markov renewal process, several measures of system validness are obtained.

(1) Transition and unchanging state probabilities
(2) Average sojourn times
(3) Reliability analysis and Average time to system failure
(4) Average Up-time for the system
(5) Average Down-time for the system
(6) Expected number of repairs of failed unit
(7) Expected number of repairs of transfer switch

1.0 Introduction: Various authors including working in the field of reliability theory have analyzed many two unit engineering systems with the assumption that both the units of the system are of similar type. But in the real practical situations it is quite reasonable to consider the standby unit as different from operative unit. The unit which is of very lower price as compared to operative unit can be considered as standby. Also it seems reasonable if we provide rest to the standby unit after a fixed amount of its operative time to make the system more reliable.

Keeping the above view, we in the present paper analyzed a two unit cold standby system in which the units are dissimilar. In the system the first unit is main operative unit which gets supremacy in operation as well as in repair and the second one is ordinary which acts as cold standby unit.

Proviso of rest is applied to the ordinary unit after its continuous operation for a constant gap of time. A transfer switch is used in changing the cold standby unit as operative. Probability that the changing state switch will be good in case of the necessity is fixed. A single repair facility is used in the system.

Using regenerative point technique with Markov renewal process, the following characteristics of reliability are obtained.

(1) Transition and unchanging state transition probabilities
(2) Average sojourn times
(3) Reliability analysis and Average time of system failing
(4) Average Up-time for the system
(5) Average Down-time of the system
(6) Probable number of repairs of failed unit
(7) Probable number of repairs of transfer switch

2.0 Features and assumptions for the Model :

The system consisting of only two dissimilar units in which first one act as priority unit and second one works as ordinary. Priority unit get preference for both working and repairing. Priority unit is more costly than ordinary unit.

Initially first unit is kept as working and the second one as cold standby.

Every unit has only two modes as Normal (N) and failing (F).

Whenever operative priority unit fails, the standby unit starts working with the help of a transfer switch. The standby starts operation if transfer switch is good otherwise there is no operation until the switch is repaired. Probability for the transferring switch will be in good position at the time of necessity is fixed.

Since ordinary unit is the cheaper one with less working efficiency so it needs rest after continuous working for a constant gap of time. The rest time of ordinary unit is also constant.

When working unit and transferring switch fails in operation, a single repairing provision is available. After repairing work of the failed unit and transfer switch works flawless.
Probability distributions of the failing units are negative exponential and probability distributions of repairing units are general. Also, the operating time and rest time probability distributions of an ordinary unit are negative exponential.

3.0 Notation and symbols:

NO : Normal unit kept as working
NS : Normal unit kept as cold standby
Fr : Failed unit under repairing
Fwr : Failed unit waiting for repairing
Nrest : Normal unit under the position of rest
Tr : Transfer switch under repair
α : Constant failure rate of operative priority unit
β : Constant failure rate of operative ordinary unit
γ : Constant rate of rest time of an ordinary unit
δ : Constant rate of operating time (after which rest is to be provide) of an ordinary unit
k(.), K(.) : probability density function and cdf of repairing time distribution of transfer switch
g(.), G(.) : probability density function and cdf of repairing time distribution of priority unit
h(.), H(.) : probability density function and cdf of repairing time distribution of ordinary unit
p(=1-q) : Probability that transfer switch will remain in good position at necessary time.

Using the above notation and symbols the the possible states of the system are given below:

Up States

$S0 \equiv$ (NO, NS) $S1 \equiv$ (Fr, NO) $S5 \equiv$ (NO, Nrest)
$S6 \equiv$ (NO, Fr)

Down States

$S2 \equiv$ (Fwr, NS, Tr) $S3 \equiv$ (Fr, Nrest) $S4 \equiv$ (Fr, Fwr)

The transitions between the different states are shown by Fig. 1. Given below

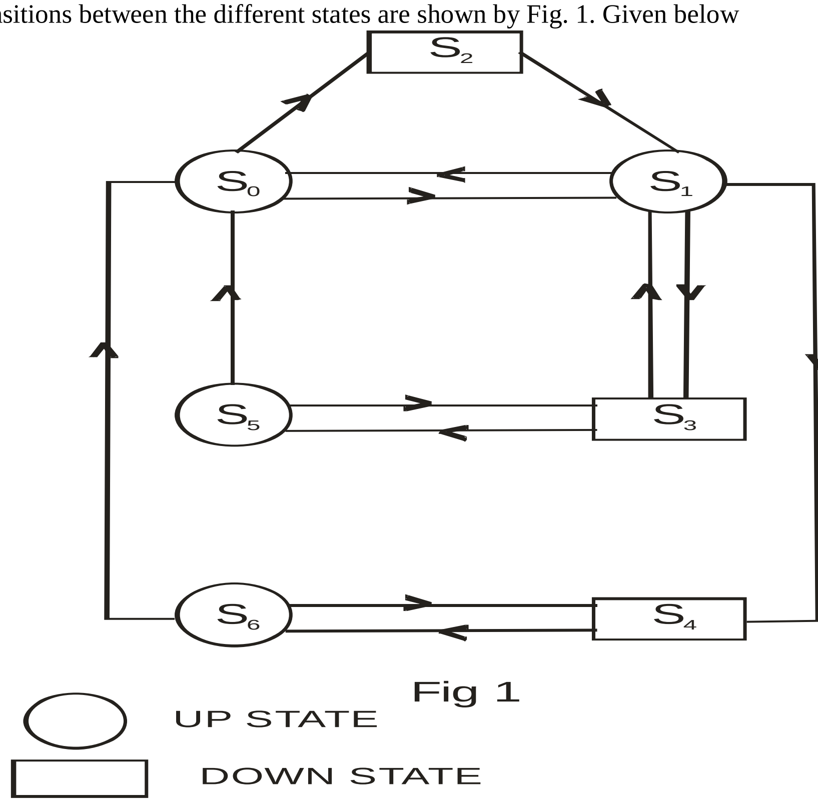

Fig 1

4.0 Probabilities for different changing states:

Let T_0 (=0), $T_1, T_2, \ldots$ be the certain duration at which the system enters the states $S_i \in E$. Let X_n denotes the state entered at epoch T_{n+1} i.e. just after the transition of T_n. Then $\{T_n, X_n\}$ constitutes a Markov-renewal process with state space E and

$$Q_{ik}(t) = \Pr[X_{n+1} = S_k, T_{n+1} - T_n \le t \mid X_n = S_i]$$

is semi Markov-Kernal over E. The stochastic matrix of the enclosed Markov chain is

$$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty)$$

By simple probabilistic consideration, the non-zero elements of $Q_{ik}(t)$ are:

$$Q_{01}(t) = p.\int_0^t \alpha e^{-\alpha u}\, du$$
$$Q_{02}(t) = q.\int_0^t \alpha e^{-\alpha u}\, du$$
$$Q_{10}(t) = \int_0^t e^{-(\beta+\delta)u}\, dG(u)$$
$$Q_{13}(t) = \delta.\int_0^t e^{-(\beta+\delta)u}\, \bar{G}(u)du$$
$$Q_{14}(t) = \int_0^t \beta e^{-(\beta+\delta)u}\, \bar{G}(u)du$$
$$Q_{21}(t) = \int_0^t dK(u)$$
$$Q_{35}(t) = \int_0^t e^{-\gamma u}\, dG(u)$$
$$Q_{46}(t) = \int_0^t dG(u)$$
$$Q_{50}(t) = \int_0^t \gamma e^{-(\alpha+\gamma)u}\, du$$
$$Q_{53}(t) = \int_0^t \alpha e^{-(\alpha+\gamma)u}\, du$$
$$Q_{60}(t) = \int_0^t e^{-\alpha u}\, dH(u)$$
$$Q_{64}(t) = \int_0^t \alpha . e^{-\alpha u}\, \bar{H}(u)du$$

$$Q^{(4)}_{16}(t) = \int_0^t \beta e^{-(\beta+\delta)u}\, \bar{G}(u)\, du. \int_u^t \frac{dG(v)}{\bar{G}(u)}$$
$$= \int_0^t dG(v) \int_0^v \beta e^{-(\beta+\delta)u}\, du, \qquad \text{(by the change of order of integration)}$$
$$= \frac{\beta}{\beta+\delta} \int_0^t (1 - e^{-(\beta+\delta)v})\, dG(v)$$

$$Q^{(1)}_{30}(t) = \frac{\gamma}{\gamma-\beta-\delta} \int_0^t (e^{-(\beta+\delta)v} - e^{-\gamma v})\, dG(v)$$

$$Q^{(1)}_{33}(t) = \frac{\delta.\gamma}{\gamma-\beta-\delta} \int_0^t (e^{-(\beta+\delta)v} - e^{-\gamma v})\, \bar{G}(v)\, du$$

$$Q^{(1,4)}_{36}(t) = \frac{\beta}{(\beta+\delta)(\gamma-\beta-\delta)} \int_0^t [(\gamma-\beta-\delta) - e^{-(\beta+\delta)w} - (\beta+\delta)e^{-\gamma w}]\, dG(w)$$

Now, we can get $q_{ij}(t)$ from $Q_{ij}(t)$ by differentiating under integral sign. Thus, we have

$$q_{01}(t) = p.\alpha e^{-\alpha t}$$
$$q_{02}(t) = q.\alpha e^{-\alpha t}$$
$$q_{10}(t) = e^{-(\beta+\delta)t}\, g(t)$$
$$q_{13}(t) = \delta e^{-(\beta+\delta)t}\, \bar{G}(t)$$
$$q_{14}(t) = \beta e^{-(\beta+\delta)t}\, \bar{G}(t)$$
$$q_{21}(t) = k(t)$$
$$q_{35}(t) = e^{-\gamma t}\, g(t)$$
$$q_{46}(t) = g(t)$$
$$q_{50}(t) = \gamma e^{-(\alpha+\gamma)t}$$

$q_{53}(t) = \alpha e^{-(\alpha+\gamma)t}$

$q_{60}(t) = e^{-\alpha t} h(t)$

$q_{64}(t) = \alpha e^{-\alpha t} \bar{H}(t)$

$q^{(4)}_{16}(t) = \frac{\beta}{\beta+\delta}(1 - e^{-(\beta+\delta)t}) g(t)$

$q^{(1)}_{30}(t) = \frac{\gamma}{\gamma-\beta-\delta}(e^{-(\beta+\delta)t} - e^{-\gamma t}) g(t)$

$q^{(1)}_{33}(t) = \frac{\delta.\gamma}{\gamma-\beta-\delta}(e^{-(\beta+\delta)t} - e^{-\gamma t}) \bar{G}(t)$

$q^{(1,4)}_{36}(t) = \frac{\beta}{(\beta+\delta)(\gamma-\beta-\delta)}[(\gamma-\beta-\delta) - e^{-(\beta+\delta)t} - (\beta+\delta)e^{-\gamma t}].g(t)$ ….(17-32)

Taking limit as $t \to \infty$, we can get the unchanging state transition p_{ij} from (1-16) . Thus

$p_{ik} = \lim_{t\to\infty} Q_{ik}(t)$

$p_{01} = p$ $p_{02} = q$

$p_{10} = \tilde{G}(\beta+\delta)$ $p_{13} = \frac{\delta}{\beta+\delta}\tilde{G}(\beta+\delta)$

$p_{14} = p^{(4)}_{16} = \frac{\beta}{\beta+\delta}[1-\tilde{G}(\beta+\delta)]$ $p_{21} = p_{46} = 1$

$p^{(1)}_{30} = \frac{\gamma}{\gamma-\beta-\delta}[\tilde{G}(\beta+\delta)-\tilde{G}(\gamma)]$

$p^{(1)}_{33} = \frac{\delta}{(\gamma-\beta-\delta)(\beta+\delta)}[(\gamma-\beta-\delta) - \gamma\tilde{G}(\beta+\delta) + (\beta+\delta)\tilde{G}(\gamma)]$

$p^{(1,4)}_{36} = \frac{\beta}{(\gamma-\beta-\delta)(\beta+\delta)}[(\gamma-\beta-\delta) - \gamma\tilde{G}(\beta+\delta) + (\beta+\delta)\tilde{G}(\gamma)]$

$p_{35} = \tilde{G}(\gamma)$ $p_{50} = \frac{\gamma}{\alpha+\gamma}$

$p_{53} = \frac{\alpha}{\alpha+\gamma}$ $p_{60} = 1 - \tilde{H}(\alpha)$

$p_{64} = \tilde{H}(\alpha)$ ….(33-48)

From the above probabilities the following relation can be verified as;

$p_{01} + p_{02} = 1$

$p_{10} + p_{13} + (p^{(4)}_{16} = p_{14}) = 1$

$p_{21} = p_{46} = 1$

$p^{(1)}_{30} + p^{(1)}_{33} + p_{35} + p^{(1,4)}_{36} = 1$

$p_{50} + p_{53} = 1$

$p_{60} + p_{64} = 1$ ….(49-55)

5.0 Average sojourn times

The average time taken by the system in a particular state Si before moving to any other state is known as average sojourn time and is defined as

$\mu_i = \int_0^\infty P[T>t]\, dt$

where T is the time of stay in state Si by the system.

To calculate average sojourn time μI in state Si, we assume that so long as the system is in state Si, it will not shift to any other state. Therefore;

$$\mu_0 = \frac{1}{\alpha}$$

$$\mu_1 = \frac{1}{\beta+\delta}[1 - \tilde{G}(\beta+\delta)]$$

$$\mu_2 = \int_0^\infty \bar{K}(t)dt = m_1 \text{ (say)}$$

$$\mu_3 = \frac{1}{\gamma}[1 - \tilde{G}(\gamma)]$$

$$\mu_4 = \int_0^\infty \bar{G}(t)dt = m_2 \text{ (say)}$$

$$\mu_5 = \frac{1}{\alpha+\gamma}$$

$$\mu_6 = \frac{1}{\alpha}[1 - \tilde{H}(\alpha)] \quad \ldots(56\text{-}62)$$

5.1 Contribution to Average Sojourn Time :

For the contribution to average sojourn time in state Si∈E and non-regenerative state occurs, before transiting to S-j∈E, i.e.,

$$m_{ij} = -\int t.q_{ij}(t)\,dt = -q'^*_{ij}(0)$$

Therefore,

$m_{01} = p.\int_0^\infty t.\alpha e^{-\alpha t}\,dt$

$m_{02} = q.\int_0^\infty t.\alpha e^{-\alpha t}\,dt$

$m_{10} = \int_0^\infty t.e^{-(\beta+\delta)t}\,dG(t)$

$m_{13} = \delta.\int_0^\infty t.e^{-(\beta+\delta)t}\,\bar{G}(t)dt$

$m_{14} = \int_0^\infty t.\beta e^{-(\beta+\delta)t}\,\bar{G}(t)dt$

$m_{21} = \int_0^\infty t.d\,K(t)$

$m_{35} = \int_0^\infty t.e^{-\gamma t}\,dG(t)$

$m_{46} = \int_0^\infty t.dG(t)$

$m_{50} = \int_0^\infty t.\gamma e^{-(\alpha+\gamma)t}\,dt$

$m_{53} = \int_0^\infty t.\alpha e^{-(\alpha+\gamma)t}\,dt$

$m_{60} = \int_0^\infty t.e^{-\alpha t}\,dH(t)$

$m_{64} = \int_0^\infty t.\alpha e^{-\alpha t}\,\bar{H}(t)dt$

$$m^{(4)}_{16} = \frac{\beta}{\beta+\delta}\int_0^\infty t.(1 - e^{-(\beta+\delta)t})\,dG(t)$$

$$m^{(1)}_{30} = \frac{\gamma}{\gamma-\beta-\delta}\int_0^\infty t.(e^{-(\beta+\delta)t} - e^{-\gamma t})\,dG(t)$$

$$m^{(1)}_{33} = \frac{\delta.\gamma}{\gamma-\beta-\delta}\int_0^\infty t.(e^{-(\beta+\delta)t} - e^{-\gamma t})\,\bar{G}(t)\,dt$$

$$m^{(1,4)}_{36} = \frac{\beta}{(\beta+\delta)(\gamma-\beta-\delta)}\int_0^\infty t.[(\gamma-\beta-\delta) - e^{-(\beta+\delta)t} - (\beta+\delta)e^{-\gamma t}]\,dG(t) \quad \ldots(63\text{-}78)$$

Hence using (63-78) the following relations can be verified as follows $m01 + m02 = p.\, {}_0\int^\infty t.\alpha e^{-\alpha t}\, dt + q.\, {}_0\int^\infty t.\alpha e^{-\alpha t}\, dt$

$$= {}_0\int^\infty t.\alpha e^{-\alpha t}\, dt = \frac{1}{\alpha} = \mu 0$$

$$m10 + m13 + m(4)16 = {}_0\int^\infty t.e^{-(\beta+\delta)t}\, dG(t) + \delta.{}_0\int^\infty t.e^{-(\beta+\delta)t}\, \bar{G}(t)dt$$

$$+ \frac{\beta}{\beta+\delta}\, {}_0\int^\infty t.(1 - e^{-(\beta+\delta)t})\, dG(t) = n1 \text{ (say)}$$

$$m21 = {}_0\int^\infty t.d\, K(t) = \mu 2 = m1$$

$$m35 + m(1)30 + m(1)33 + m(1,4)36 = {}_0\int^\infty t.e^{-\gamma t}\, dG(t)$$

$$+ \frac{\gamma}{\gamma-\beta-\delta}\, {}_0\int^\infty t.(e^{-(\beta+\delta)t} - e^{-\gamma t})\, dG(t)$$

$$+ \frac{\delta.\gamma}{\gamma-\beta-\delta}\, {}_0\int^\infty t.(e^{-(\beta+\delta)t} - e^{-\gamma t})\, \bar{G}(t)\, dt$$

$$+ \frac{\beta}{(\beta+\delta)(\gamma-\beta-\delta)}\, {}_0\int^\infty t.[(\gamma-\beta-\delta) - e^{-(\beta+\delta)t} - (\beta+\delta)e^{-\gamma t}]dG(t)$$

$= n2$ (say)

$$m46 = {}_0\int^\infty t.dG(t) = \mu 4 = m2$$

$$m50 + m53 = {}_0\int^\infty t.\gamma e^{-(\alpha+\gamma)t}\, dt + {}_0\int^\infty t.\alpha e^{-(\alpha+\gamma)t}\, dt$$

$$= \frac{1}{\alpha+\gamma} = \mu 5$$

$$m60 + m64 = {}_0\int^\infty t.e^{-\alpha t}\, dH(t) + {}_0\int^\infty t.\alpha e^{-\alpha t}\, \bar{H}(t)dt$$

$$= \frac{1}{\alpha}[1 - \tilde{H}(\alpha)] = \mu 6 \qquad \text{….(79-85)}$$

6.0 Reliability and Average time for system failing :

To find reliability of this system we regard all the failed states as absorbing states. If Ti is time for system failing when it starts working from state Si, reliability of this system is given by

$Ri(t) = Pr[Ti>t]$ Using the probabilistic arguments considering reliability the following recursive relations can be easily obtained.

$R0(t) = Z0(t) + q01(t)©R1(t) + q02(t)©R2(t)$

$R1(t) = Z1(t) + q10(t)©R0(t) + q13(t)©R3(t)$

$R2(t) = Z2(t) + q21(t)©R1(t)$

$R3(t) = Z3(t) + q(1)30(t)©R0(t) + q(1)33(t)©R3(t) + q35(t)©R5(t)$ $R5(t) = Z5(t) + q50(t)©R0(t) + q53(t)©R3(t)$

$R6(t) = Z6(t) + q60(t)©R0(t)$ ….(86-91)

where

$Z0(t) = e^{-\alpha t}$ $\qquad Z1(t) = e^{-(\beta+\delta)t}\bar{G}(t)$

$Z2(t) = \bar{K}(t)$ $\qquad Z3(t) = \frac{\gamma}{\gamma-\beta-\delta}[\gamma e^{-(\beta+\delta)t} - (\beta+\delta)e^{-\gamma t})]\bar{G}(t)$

$Z_5(t) = e^{-(\alpha+\gamma)t}$ $Z_6(t) = e^{-\alpha t}\overline{H}(t)$

After applying Laplace transformations in equations (86-91) one gets

$R^*_0(s) = Z^*_0(s) + q^*_{01}(s).R^*_1(s) + q^*_{02}(s).R^*_2(s)$

$R^*_1(s) = Z^*_1(s) + q^*_{10}(s).R^*_0(s) + q^*_{13}(s).R^*_3(s)$

$R^*_2(s) = Z^*_2(s) + q^*_{21}(s).R^*_1(s)$

$R^*_3(s) = Z^*_3(s) + q^{*(1)}_{30}(s).R^*_0(s) + q^{*(1)}_{33}(s).R^*_3(s)$
$\quad + q^*_{35}(s).R^*_5(s)$

$R^*_5(s) = Z^*_5(s) + q^*_{50}(s).R^*_0(s) + q^*_{53}(s).R^*_3(s)$

$R^*_6(s) = Z^*_6(s) + q^*_{60}(s).R^*_0(s)$ ….(92-97)

Solving the above equations (92-97) for $R^*_0(s)$, we get

$R^*_0(s) = N_1(s)/D_1(s)$ ….(98)

where

$N_1(s) = (1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})(Z^*_0 + Z^*_1q^*_{01} + Z^*_1q^*_{02}q^*_{21}$

$+ Z^*_2q^*_{02}) + q^*_{13}(Z^*_3 + Z^*_5q^*_{35})(q^*_{01} + q^*_{02}q^*_{21})$ ….(99)

and

$D_1(s) = (1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})(1 - q^*_{01}q^*_{10} - q^*_{02}q^*_{21}q^*_{10})$

$- q^*_{13}(q^{*(1)}_{30} + q^*_{35}q^*_{50})(q^*_{01} + q^*_{02}q^*_{21})$ ….(100)

Now, by using inverse Laplace transformations in result (98), we can get system reliability for well-known failing time distributions. Also we can get Average time for system failing (ATSF) when the system begins working from initial state S_0 as

$E(T_0) = \lim_{s\to 0} R^*_0(s) = N_1(0)/D_1(0)$ ….(101)

Now,

$Z^*_0(0) = \int_0^\infty e^{-\alpha t}\,dt = \frac{1}{\alpha} = \mu_0$

$Z^*_1(0) = \int_0^\infty e^{-(\beta+\delta)t}\,\overline{G}(t)dt$

$= \frac{1}{\beta+\delta}[1 - \tilde{G}(\beta+\delta)] = \mu_1$

$Z^*_2(0) = \int_0^\infty \overline{K}(t)dt = m_1$

$Z^*_3(0) = \frac{\gamma}{\gamma-\beta-\delta}\int_0^\infty [\gamma e^{-(\beta+\delta)t} - (\beta+\delta)e^{-\gamma t}]\,\overline{G}(t)dt = L$ (say)

$Z_5(t) = \int_0^\infty e^{-(\alpha+\gamma)t}dt = \frac{1}{\alpha+\gamma} = \mu_5$

$Z_6(t) = \int_0^\infty e^{-\alpha t}\overline{H}(t)\,dt = \frac{1}{\alpha}[1 - \tilde{H}(\alpha)] = \mu_6$ ….(102-107)

Therefore,

$N_1(0) = (1 - p^{(1)}_{33} - p_{35}p_{53})(\mu_0 + \mu_1 + m_1p_{02}) + p_{13}(L + \mu_5p_{35})$ ….(108)

and

$D_1(0) = p_{14}(1 - p^{(1)}_{33} - p_{35}p_{53}) + p_{13}p^{(1,4)}_{36}$ ….(109)

Hence, $E(T_0) = N_1/D_1$ ….(110)

where N1 and D1 are same as in (108) and (109).

7.0 Average up-time of system:

Let Ai(t) represent the probability that starting from state Si the system will remains up at fixed time t. Using probabilistic arguments the following recursive relations can be easily obtained

$$A_0(t) = M_0(t) + q_{01}(t)\copyright A_1(t) + q_{02}(t)\copyright A_2(t)$$
$$A_1(t) = M_1(t) + q_{10}(t)\copyright A_0(t) + q_{13}(t)\copyright A_3(t) + q^{(4)}_{16}(t)\copyright A_6(t)$$
$$A_2(t) = q_{21}(t)\copyright A_1(t)$$
$$A_3(t) = M_3(t) + q^{(1)}_{30}(t)\copyright A_0(t) + q^{(1)}_{33}(t)\copyright A_3(t) + q_{35}(t)\copyright A_5(t) + q^{(1,4)}_{36}(t)\copyright A_6(t)$$
$$A_4(t) = q_{46}(t)\copyright A_6(t)$$
$$A_5(t) = q_{50}(t)\copyright A_0(t) + q_{53}(t)\copyright A_3(t)$$
$$A_6(t) = M_6(t) + q_{60}(t)\copyright A_0(t) + q_{64}(t)\copyright A_4(t) \qquad \text{....(111-117)}$$

where

$$M_0(t) = e^{-\alpha t}$$
$$M_1(t) = e^{-(\beta+\delta)t}\,\overline{G}(t)$$
$$M_3(t) = \frac{\gamma}{\gamma-\beta-\delta}\, e^{-(\beta+\delta+\gamma)t}\,\overline{G}(t)$$
$$M_6(t) = e^{-\alpha t}\,\overline{H}(t)$$

After applying laplace transform in equations (111-117) one gets

$$A^*_0(s) = M^*_0(s) + q^*_{01}(s).A^*_1(s) + q^*_{02}(s).A^*_2(s)$$
$$A^*_1(s) = M^*_1(s) + q^*_{10}(s).A^*_0(s) + q^*_{13}(s).A^*_3(s) + q^{*(4)}_{16}(s).A^*_6(s)$$
$$A^*_2(s) = q^*_{21}(s).A^*_1(s)$$
$$A^*_3(s) = M^*_3(s) + q^{*(1)}_{30}(s).A^*_0(s) + q^{*(1)}_{33}(s).A^*_3(s) + q^*_{35}(s).A^*_5(s) + q^{*(1,4)}_{36}(s).A^*_6(s)$$
$$A^*_4(s) = q^*_{46}(s).A^*_6(s)$$
$$A^*_5(s) = q^*_{50}(s).A^*_0(s) + q^*_{53}(s).A^*_3(s)$$
$$A^*_6(s) = M^*_6(s) + q^*_{60}(s).A^*_0(s) + q^*_{64}(s).A^*_4(s) \qquad \text{....(118-124)}$$

Now, solving for point wise availability A*0(s), one gets

$$A^*_0(s) = \frac{N_2(s)}{D_2(s)} \qquad \text{....(125)}$$

where

$$N_2(s) = (1 - q^*_{46}q^*_{64})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})[M^*_0 + M^*_1(q^*_{01} + q^*_{02}q^*_{21})] + q^*_{13}(q^*_{01} + q^*_{02}q^*_{21})$$
$$[(1 - q^*_{46}q^*_{64})q^*_{35}M^*_5 + q^{*(1,4)}_{36}M^*_6] + M^*_6 q^{*(4)}_{16}(q^*_{01} + q^*_{02}q^*_{21})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53}) \qquad \text{....(126)}$$

and

$$D_2(s) = (1 - q^*_{46}q^*_{64})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})[1 - q^*_{01}(q^*_{01} + q^*_{02}q^*_{21})] - q^*_{13}(q^*_{01} + q^*_{02}q^*_{21})[(1 - q^*_{46}q^*_{64})$$
$$.(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53}) - q^{*(1,4)}_{36}(1 - q^*_{60} - q^*_{46}q^*_{64})]$$
$$- (q^*_{01} + q^*_{02}q^*_{21})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})q^{*(4)}_{16}q^*_{60} \qquad \text{....(127)}$$

Now,

$$D_2(0) = (1 - p_{46}p_{64})(1 - p^{(1)}_{33} - p_{35}p_{53})[1 - p_{01}(p_{01} + p_{02}p_{21})] - p_{13}(p_{01} + p_{02}p_{21})[p_{13}p^{(1,4)}_{36}(1 - p_{60} - p_{46}p_{64})] - (1 - p^{(1)}_{33} - p_{35}p_{53})]p^{(4)}_{16}p_{60}$$
$$= (1 - p_{64})(1 - p^{(1)}_{33} - p_{35}p_{53})(1 - p_{10} - p_{13}) + [p_{13}p^{(1,4)}_{36}(1 - p_{60} - p_{64})] - (1 - p^{(1)}_{33} - p_{35}p_{53})p^{(4)}_{16}p_{60}$$
$$= (1 - p^{(1)}_{33} - p_{35}p_{53})[p^{(4)}_{16}p_{60} - (1 - p^{(1)}_{33} - p_{35}p_{53})]p^{(4)}_{16}p_{60} = 0 \qquad \text{....(128)}$$

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Now, for obtaining D'2(0) the coefficients of mij's in D'2(0) are,

m01 $\rightarrow$ p60(1 – p(1)33 – p35p53)
m02 $\rightarrow$ p60(1 – p(1)33 – p35p53)
m10 $\rightarrow$ p60(1 – p(1)33 – p35p53)
m13 $\rightarrow$ p60(1 – p(1)33 – p35p53)
m(4)16 $\rightarrow$ p60(1 – p(1)33 – p35p53)
m21 $\rightarrow$ p02p60(1 – p(1)33 – p35p53)
m(1)30 $\rightarrow$ 0
m(1)33 $\rightarrow$ 0
m35 $\rightarrow$ 0
m(1,4)36 $\rightarrow$ 0
m46 $\rightarrow$ p(4)16p64(1 – p(1)33 – p35p53) + p13p(1,4)36p64
m50 $\rightarrow$ 0
m53 $\rightarrow$ 0
m60 $\rightarrow$ p13p(1,4)36 + p(4)16(1 – p(1)33 – p35p53)
m64 $\rightarrow$ p13p(1,4)36 + p(4)16(1 – p(1)33 – p35p53)

Thus,

D'2(0) = p60(1 – p(1)33 – p35p53)(μ0 + n1 + m1p02) + [p13p(1,4)36
 + p(4)16(1 – p(1)33 – p35p53)(μ6 + m2p64) ….(129)

Also,

M*0(0) = 0$\int\infty$e-αtdt = 1/α = μ0

M*1(0) = 0$\int\infty$e-(β+δ)t $\overline{G}$ (t) dt = μ1

M*3(0) = $\frac{\gamma}{\gamma-\beta-\delta}$ 0$\int\infty$(e-(β+δ)t - e-γt). $\overline{G}$ (t)dt = $\frac{(\mu_1-\mu_3)\gamma}{(\gamma-\beta-\delta)}$ = L1 (say)

M*6(0) = 0$\int\infty$e-αt $\overline{H}$ (t) dt = μ6

And then

N2(0) = (1 - p(1)33 - p35p53)[p60(μ0 + μ1) + p(4)16μ6]
 + p13[p(1,4)36μ6 + p60(p35μ5 + L1)] ….(130)

Therefore,

A0 = lim U0(t) = lim s.U*0(s)
 t$\rightarrow\infty$ s$\rightarrow$0
 = lim N2(s)/D2(s) = N2/D2 ….(131)
 s$\rightarrow$0

Where N2 and D2 are identical as in equations (130) and (129) respectively.

8.0 Average Down-time for system:

Let Bi(t) denote probability for starting from regenerative down state Sj the system shall remain down at fixed time t. Using probabilistic arguments we can get relations given below also known as recursive relations.

B0(t) = q01(t)©B1(t) + q02(t)©B2(t)
B1(t) = q10(t)©B0(t) + q13(t)©B3(t) + q(4)16(t)©B6(t)
B2(t) = M2(t) + q21(t)©B1(t)
B3(t) = M3(t) + q30(t)©B0(t) + q(1)33(t)©B3(t) + q35(t)©B5(t)
 + q(1,4)36(t)©B6(t)
B4(t) = q46(t)©B6(t)
B5(t) = q50(t)©B0(t) + q53(t)©B3(t)
B6(t) = q60(t)©B0(t) + q64(t)©B4(t) ….(132-138)

where

M2(t) = $\overline{K}$ (t)

M3(t) = e-γt $\overline{G}$ (t)

After applying Laplace transforms in equations (132-138) one gets

$B^*_0(s) = q^*_{01}(s).B^*_1(s) + q^*_{02}(s).B^*_2(s)$
$B^*_1(s) = q^*_{10}(s).B^*_0(s) + q^*_{13}(s).B^*_3(s) + q^{*(4)}_{16}(s).B^*_6(s)$
$B^*_2(s) = M^*_2(s) + q^*_{21}(s).B^*_1(s)$
$B^*_3(s) = M^*_3(s) + q^*_{30}(s).B^*_0(s) + q^{*(1)}_{33}(s).B^*_3(s)$
$\quad + q^*_{35}(s).B^*_5(s) + q^{*(1,4)}_{36}(s).B^*_6(s)$
$B^*_4(s) = q^*_{46}(s).B^*_6(s)$
$B^*_5(s) = q^*_{50}(s).B^*_0(s) + q^*_{53}(s).B^*_3(s)$
$B^*_6(s) = q^*_{60}(s).B^*_0(s) + q^*_{64}(s).B^*_4(s)$ ….(139-145)

Now, solving the above equations for $B^*_0(s)$, by removing the arguments 's' for brevity, one gets

$$B^*_0(s) = \frac{N_3(s)}{D_3(s)} \quad ….(146)$$

where

$N_3(s) = (1 - q^*_{46}q^*_{64})(1 - q^{*(1)}_{33} - q^*_{35}q^*_{53})M^*_2q^*_{02}$
$\quad + q^*_{13}(q^*_{01} + q^*_{02}q^*_{21})(1 - q^*_{46}q^*_{64})M^*_3$ ….(147)

and $D_3(s)$ is same as $D_2(s)$ in equation (127).

Now,

$M^*_2(0) = {}_0\int^\infty \bar{K}(t)\,dt = \mu_2 = m_1$

$M^*_3(0) = {}_0\int^\infty e^{-\gamma t}\,\bar{G}(t)\,dt = \mu_3$

Then,

$N_3(0) = p_{60}[(1 - p^{(1)}_{33} - p_{35}p_{53})p_{02}m_1 + p_{13}\mu_3]$ ….(148)

Therefore,

$$B_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s.B^*_0(s)$$

$$= \lim_{s\to 0} N_3(s)/D_3(s) = N_3(0)/D'_3(0) = N_3/D_3 \quad ….(149)$$

where N_3 is same as in (148) and D_3 is as D_2 in (129).

9.0 Probable Number of repairs for failing unit :

Let we define, $V_i(t)$ as the probable number of repairs completed by the repairman in (0,t] subject to the condition that the system initially started from recreating state S_i at t=0. Then following results for $V_i(t)$'s can be calculated as;

$V_0(t) = Q_{01}(t)\$V_1(t) + Q_{02}(t)\$V_2(t)$
$V_1(t) = Q_{10}(t)\$[1 + V_0(t)] + Q_{13}(t)\$V_3(t) + Q^{(4)}_{16}(t)\$[1+V_6(t)]$
$V_2(t) = Q_{21}(t)\$V_1(t)$
$V_3(t) = Q^{(1)}_{30}(t)\$[1 + V_0(t)] + Q^{(1)}_{33}(t)\$V_3(t) + Q_{35}(t)\$[1 + V_5(t)]$
$\quad + Q^{(1,4)}_{36}(t)\$[1+ V_6(t)]$
$V_4(t) = Q_{46}(t)\$[1 + V_6(t)]$
$V_5(t) = Q_{50}(t)\$V_0(t) + Q_{53}(t)\$V_3(t)$
$V_6(t) = Q_{60}(t)\$[1 + V_0(t)] + Q_{64}(t)\$V_4(t)$ ….(150-156)

Now after applying laplace stieltjes transformations in above equations (150-156), we get

$\tilde{V}_0(s) = \tilde{Q}_{01}(s).\tilde{V}_1(s) + \tilde{Q}_{02}(s).\tilde{V}_2(s)$

$\tilde{V}_1(s) = \tilde{Q}_{10}(s).[1 + \tilde{V}_0(s)] + \tilde{Q}_{13}(s).\tilde{V}_3(s) + \tilde{Q}^{(4)}_{16}(s).[1 + \tilde{V}_6(s)]$

$\tilde{V}_2(s) = \tilde{Q}_{21}(s).\tilde{V}_1(s)$

$$\tilde{V}_3(s) = \tilde{Q}^{(1)}_{30}(s).[1 + \tilde{V}_0(s)] + \tilde{Q}^{(1)}_{33}(s).\tilde{V}_3(s) + \tilde{Q}_{35}(s).[1 + \tilde{V}_5(s)] + \tilde{Q}^{(1,4)}_{36}(s).[1 + \tilde{V}_6(s)]$$

$$\tilde{V}_4(s) = \tilde{Q}_{46}(s).[1 + \tilde{V}_6(s)]$$

$$\tilde{V}_5(s) = \tilde{Q}_{50}(s).\tilde{V}_0(s) + \tilde{Q}_{53}(s).\tilde{V}_3(s)$$

$$\tilde{V}_6(s) = \tilde{Q}_{60}(s).[1 + \tilde{V}_0(s)] + \tilde{Q}_{64}(s).\tilde{V}_4(s) \quad ….(157\text{-}163)$$

And the solution of $\tilde{V}_0(s)$ may be expressed as

$$\tilde{V}_0(s) = N_4(s)/D_4(s) \quad ....(164)$$

where

$$N_4(s) = (1 - \tilde{Q}_{46}\tilde{Q}_{64})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53})(\tilde{Q}_{10} + \tilde{Q}^{(4)}_{16})(\tilde{Q}_{01} + \tilde{Q}_{02}\tilde{Q}_{21}) + \tilde{Q}^{(4)}_{16}(\tilde{Q}_{46}\tilde{Q}_{64} + \tilde{Q}_{60})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53})(\tilde{Q}_{01} + \tilde{Q}_{02}\tilde{Q}_{21}) + \tilde{Q}_{01}\tilde{Q}_{13}[(\tilde{Q}^{(1)}_{30} + \tilde{Q}_{35} + \tilde{Q}^{(1,4)}_{36})(1 - \tilde{Q}_{46}\tilde{Q}_{64}) + \tilde{Q}^{(1,4)}_{36}(\tilde{Q}_{46}\tilde{Q}_{64} + \tilde{Q}_{60})] \quad ….(165)$$

and

$$D_4(s) = (1 - \tilde{Q}_{46}\tilde{Q}_{64})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53})[1 - \tilde{Q}_{10}(\tilde{Q}_{01} + \tilde{Q}_{02}\tilde{Q}_{21})] - \tilde{Q}_{13}(\tilde{Q}_{01} + \tilde{Q}_{02}\tilde{Q}_{21})[(1 - \tilde{Q}_{46}\tilde{Q}_{64})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53}) - \tilde{Q}^{(1,4)}_{36}(1 - \tilde{Q}_{60} - \tilde{Q}_{46}\tilde{Q}_{64})] - (\tilde{Q}_{01} + \tilde{Q}_{02}\tilde{Q}_{21})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53})\tilde{Q}^{(4)}_{16}\tilde{Q}_{60} \quad ….(166)$$

Now,

N4(0) = (1 - p(1)33 - p35p53)(p60(1 – p13) + p(4)16)

+ p01p13[p60(1 – p(1)33) + p(1,4)36] ….(167)

Now, probable number of repairs for failing unit is given by

$$V0 = \lim_{t \to \infty} [V0(t)/t] = \lim_{s \to 0} s \tilde{V}0(s) = N4/D4 \quad ....(168)$$

where N4 is identical as (164) and D4 is identical as D2 in (129).

10. PROBABLE number of repairs FOR transfer switch

Let we define, V'i(t) as the probable number of repairs completed by the repairman for transferring switch in (0,t] subject to the condition that the system initially started from recreating state Si at t=0. Then following results among V'i(t)'s can be calculated as;

V'0(t) = Q01(t)$V'1(t) + Q02(t)$V'2(t)

V'1(t) = Q10(t)$V'0(t) + Q13(t)$V'3(t) + Q(4)16(t)$V'6(t)

V'2(t) = Q21(t)$V'1(t)

V'3(t) = Q(1)30(t)$V'0(t) + Q(1)33(t)$V'3(t) + Q35(t)$V'5(t)

+ Q(1,4)36(t)$V'6(t)

V'4(t) = Q46(t)$V'6(t)

V'5(t) = Q50(t)$V'0(t) + Q53(t)$V'3(t)

V'6(t) = Q60(t)$V'0(t) + Q64(t)$V'4(t) ….(169-175)

Taking laplace stiltjes transformations in above equations (169-175) we get,

$\tilde{V}'0(s) = \tilde{Q}01(s).\tilde{V}'1(s) + \tilde{Q}02(s).\tilde{V}'2(s)$

$\tilde{V}'1(s) = \tilde{Q}10(s).\tilde{V}'0(s)] + \tilde{Q}13(s).\tilde{V}'3(s) + \tilde{Q}(4)16(s)\tilde{V}'6(s)$

$\tilde{V}'2(s) = \tilde{Q}21(s).\tilde{V}'1(s)$

$\tilde{V}'3(s) = \tilde{Q}(1)30(s).\tilde{V}'0(s) + \tilde{Q}(1)33(s).\tilde{V}'3(s) + \tilde{Q}'35(s).\tilde{V}'5(s)$

$+ \tilde{Q}(1,4)36(s).\tilde{V}'6(s)$

$\tilde{V}'4(s) = \tilde{Q}46(s).\tilde{V}'6(s)$

$\tilde{V}'5(s) = \tilde{Q}50(s).\tilde{V}'0(s) + \tilde{Q}53(s).\tilde{V}'3(s)$

$\tilde{V}'6(s) = \tilde{Q}60(s).\tilde{V}'0(s) + \tilde{Q}64(s).\tilde{V}'4(s)$ ….(176-182)

And the solution of $\tilde{V}0(s)$ may be expressed as:

$$\tilde{V}_0(s) = N_5(s)/D_5(s) \quad ....(183)$$

where

$$N_5(s) = (1 - \tilde{Q}_{46}\tilde{Q}_{64})(1 - \tilde{Q}^{(1)}_{33} - \tilde{Q}_{35}\tilde{Q}_{53})\tilde{Q}_{02}\tilde{Q}_{21} \quad ....(184)$$

and $D_5(s)$ is same as in $D_4(s)$ in (166).
Now,

$$N_5(0) = p_{60}p_{02}(1 - p^{(1)}_{33} - p_{35}p_{53}) \quad ....(185)$$

Therefore, in unchanging state the probable number of repairs of transferring switch is given by:

$$V_0 = \lim_{t \to \infty} [V_0(t)/t] = \lim_{s \to 0} s\,\tilde{V}_0(s) = N_5/D_5 \quad ....(186)$$

where N_5 is identical to (185) and D_4 is identical to D_2 in (129).

10.0 Conclusion:

Mathematical results obtained in this study is useful for finding average time of system failure, average time spent by a repairman and probable number of repairs required, when the cold standby used in a production house is of low cost.

11.0 References

1. Agarwal, S.C., Mamta S., and Shikha, B. (2010): Reliability characteristic of cold-standby redundant system, International Journal of Research and Reviews in Applied Sciences, 3(2), p. 193-199.
2. Agnihotri, R.K, (2011) "Stochastic behaviour of an engineering system with guarantee and warrantee period", J. Scientific & tech. Res., Vol. 1, Issue 2, 26-33.
3. Agnihotri, R.K, Sharma, G. and Jain, S. (2010) : "Reliability analysis of a system of wheels in a four wheeler", Int. J. Agricult. Stat. Sci., Vol. 6, No. 1, , 259-268.
4. EL-Damcese, M.A. and Tamraz, N.S. (2012) : Analysis of a parallel redundant system with different failure mode, JRSS, Vol. 5, Issue (1), pp. 95-106.
5. Singh, S.K. and Sheeba G. Nair (1999) : "Stochastic analysis of a milk pack machine subject to shut-off rules", Opsearch, Vol. 36, No.3, 231-241.