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STOCHASTIC BEHAVIOUR OF A REDUNDANT SYSTEM WITH INTERCHANGEMENT FACILITY IN STANDBY

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Abstract: The present paper provides the stochastic analysis of an engineering system consisting of two identical electronic machines. In the system working machine needs rest after a fixed amount of continuous working, so the system is provided with the facility of interchangement of warm standby in to cold and vice versa after a random amount of time. Using regenerative point techniques various reliability characteristics which are useful to system designers are obtained.

Keywords: interchangement, symbols, Sojourn Time.

1.0 Introduction: Many researchers working in the field of engineering reliability have studied many system models by using different sets of assumptions. Most of them assumed that warm standby remains in the position of warm/operative throughout the system. Since warm standby can also fail and it is having some non-zero running cost so for improving its life, it is beneficial to interchange the warm standby into cold and vice-versa after a random amount of time.

Keeping the above view, we in the present paper provides the reliability analysis of a production unit consisting of two identical electronic machines with the facility of interchangement of warm standby into cold.

Using regenerative point technique with Markov renewal process various reliability measures which are useful to industrial development are obtained.

2.0 Model description and assumptions

(1) The system consists of only two electronic machines which are identical. Each of the machines is having two modes as Normal and Complete failure.
(2) Initially one electronic machine is operative and other is kept as warm standby.
(3) The warm standby becomes a cold standby after a random amount of time and vice-versa to provide the rest for improving its life. Upon failure of an operative electronic machine the warm standby comes into help of a automatic electronic device "AED" instantaneous legible random amount of time to start operation.
(4) A single repair facility is used with discipline "FCFS" in the system and once the repairman enters the system he will complete all the jobs related to repair.
(5) The probability that a repaired electronic machine will be placed as warm standby or cold standby is fixed.
(6) The failure time distribution for both the electronic machines is exponential, while the distribution of time to complete repair is arbitrary.
(7) The distribution of time to interchange warm standby into cold standby and vice-versa are arbitrary.
(8) The distribution of time to start operation by the electronic machine kept as cold standby is negative exponential.

3.0 Notation and symbols:

N_O	:	Normal electronic machine kept as operative
N_{WS}	:	Normal electronic machine kept as warm standby
N_{CS}	:	Normal electronic machine kept as cold standby
N_{CSS}	:	Normal electronic machine kept as cold standby being switched on for operation
F_r	:	Failed electronic machine under repair

F_{wr}	:	Failed electronic machine waiting for repair
α	:	Constant failure rate of an operative electronic machine
β	:	Constant failure rate of an electronic machine kept as warm standby
γ	:	Constant rate to interchange cold standby into warm standby
δ	:	Constant rate of time for an electronic machine kept as cold standby to become operative by the help of "AED"
g(.), G(.)	:	pdf and cdf of time to repair a failed electronic machine
h(.), H(.)	:	pdf and cdf of time after which warm standby electronic machine interchange as cold standby
p	:	Probability that the repaired electronic machine will be placed as cold standby
q	:	Probability that the repaired electronic machine will be placed as warm standby

Using the above notation and symbols the following are the possible states of the system:

Up States

$S_0 \equiv (N_O, N_{WS})$ $S_1 \equiv (N_O, F_r)$ $S_2 \equiv (N_O, N_{CS})$

Down States

$S_3 \equiv (F_r, F_{wr})$ $S_4 \equiv (F_r, N_{CSS})$ $S_5 \equiv (N_{CS}, N_{CSS})$

$S_6 \equiv (N_{WS}, N_{CSS})$

The possible transitions between the above states are shown in Fig. 1.

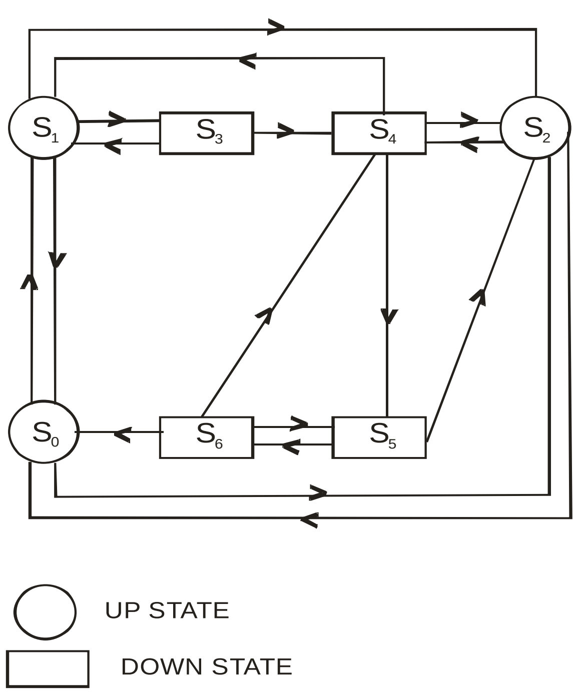

Fig 1

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4.0 Transition probabilities:

Let T0 (=0), T1,T2,.... be the epochs at which the system enters the states Si $\in$ E. Let Xn denotes the state entered at epoch Tn+1 i.e. just after the transition of Tn. Then {Tn,Xn} constitutes a Markov-renewal process with state space E and

$Q_{ik}(t) = Pr[X_{n+1} = S_k, T_{n+1} - T_n \le t \mid X_n = S_i]$

is semi Markov-Kernal over E. The stochastic matrix of the embedded Markov chain is

$$P = p_{ik} = \lim_{t\to\infty} Q_{ik}(t) = Q(\infty)$$

By simple probabilistic consideration, the non-zero elements of Qik(t) are:

$Q_{01}(t) = (\alpha+\beta).{}_0\int^t e^{-(\alpha+\beta)u}\,\bar{H}(u)du$ $\quad Q_{02}(t) = {}_0\int^t e^{-(\alpha+\beta)u}\,h(u)du$

$Q_{10}(t) = q.{}_0\int^t e^{-\alpha u}\,g(u)du$ $\quad Q_{12}(t) = p.{}_0\int^t e^{-\alpha u}\,g(u)du$

$Q_{13}(t) = {}_0\int^t \alpha e^{-\alpha u}\,\bar{G}(u)du$ $\quad Q_{20}(t) = {}_0\int^t \gamma e^{-(\alpha+\gamma)u}\,du$

$Q_{24}(t) = {}_0\int^t \alpha e^{-(\alpha+\gamma)u}\,du$ $\quad Q_{31}(t) = q.{}_0\int^t g(u)du$

$Q_{34}(t) = p.{}_0\int^t g(u)du$ $\quad Q_{42}(t) = q.{}_0\int^t e^{-\delta u}\,g(u)du$

$Q_{45}(t) = p.{}_0\int^t e^{-\delta u}\,g(u)du$ $\quad Q_{52}(t) = {}_0\int^t \delta e^{-(\delta+\gamma)u}\,du$

$Q_{56}(t) = {}_0\int^t \gamma e^{-(\delta+\gamma)u}\,du$ $\quad Q_{64}(t) = {}_0\int^t \beta e^{-(\beta+\delta)u}\,\bar{H}(u)du$

$Q_{65}(t) = {}_0\int^t e^{-(\beta+\delta)u}\,h(u)du$

$$Q^{(3)}_{11}(t) = {}_0\int^t \alpha e^{-\alpha u}\,\bar{G}(u)\,du.\ q.\int_u^t \frac{dG(v)}{\bar{G}(u)}$$

$= q.{}_0\int^t \alpha.dG(v)\ {}_0\int^v e^{-\alpha u}du$, (by the change of order of integration)

$= q.{}_0\int^t (1 - e^{-\alpha v})\,g(v)dv$

$Q^{(3)}_{14}(t) = p.{}_0\int^t (1 - e^{-\alpha v})\,g(v)dv$

$Q^{(1)}_{40}(t) = q.{}_0\int^t A(v).g(v)dv$

$Q^{(1)}_{42}(t) = p.{}_0\int^t A(v).g(v)dv$

$Q^{(1)}_{43}(t) = \alpha.{}_0\int^t A(v).\bar{G}(v)\,dv$

$Q^{(0)}_{61}(t) = (\alpha+\beta).{}_0\int^t e^{-\beta v}A(v).\bar{H}(v)dv$

$Q^{(0)}_{62}(t) = {}_0\int^t e^{-\beta v}A(v).h(v)dv$ ....(1-22)

where

$$A(v) = \frac{\delta}{\delta-\alpha}(e^{-\alpha v}-e^{-\delta v})$$

$Q_{(1,3)41}(t) = \int_0^t \delta e^{-\delta u}\, \overline{G}(u)\, du. \int_u^t \alpha e^{-\alpha(v-u)} \frac{\overline{G}(v)}{\overline{G}(u)} dv \int_v^t q \frac{dG(w)}{\overline{G}(v)} dv$

$= q.\int_0^t \alpha\delta.dG(w) \int_0^w e^{-\alpha v} dv, \int_0^v e^{-(\delta-\alpha)u} du$

$= q.\frac{1}{\delta-\alpha} \int_0^t [(\delta-\alpha)-\delta e^{-\alpha w}+\alpha e^{-\alpha w}] g(w)dw$

$Q_{(1,3)44}(t) = p.\frac{1}{\delta-\alpha} \int_0^t [(\delta-\alpha)-\delta e^{-\alpha w}+\alpha e^{-\alpha w}] g(w)dw$ ….(23-24)

Taking limit as $t \to \infty$, the steady state transition p_{ij} can be obtained from (1-24) . Thus

$p_{ik} = \lim_{t\to\infty} Q_{ik}(t)$

$p_{01} = 1 - \tilde{H}(\alpha+\beta)$ $\quad$ $p_{02} = \tilde{H}(\alpha+\beta)$

$p_{10} = q.\tilde{G}(\alpha)$ $\quad$ $p_{12} = p.\tilde{G}(\alpha)$

$p_{13} = 1 - \tilde{G}(\alpha)$ $\quad$ $p_{(3)11} = q.[1 - \tilde{G}(\alpha)]$

$p_{(3)14} = p.[1 - \tilde{G}(\alpha)]$ $\quad$ $p_{20} = \frac{\gamma}{\alpha+\gamma}$

$p_{24} = \frac{\alpha}{\alpha+\gamma}$ $\quad$ $p_{31} = q$

$p_{34} = p$ $\quad$ $p_{42} = q.\tilde{G}(\delta)$

$p_{45} = p.\tilde{G}(\delta)$ $\quad$ $p_{(1)40} = q.\frac{\delta}{\delta-\alpha}[\tilde{G}(\alpha) - \tilde{G}(\delta)]$

$p_{(1)42} = p.\frac{\delta}{\delta-\alpha}[\tilde{G}(\alpha) - \tilde{G}(\delta)]$

$p_{(1)43} = \frac{1}{\delta-\alpha}[(\delta-\alpha)-\delta\tilde{G}(\alpha)+\alpha\tilde{G}(\delta)]$

$p_{(1,3)41} = q.\frac{1}{\delta-\alpha}[(\delta-\alpha)-\delta\tilde{G}(\alpha)+\alpha\tilde{G}(\delta)]$

$p_{(1,3)44} = p.\frac{1}{\delta-\alpha}[(\delta-\alpha)-\delta\tilde{G}(\alpha)+\alpha\tilde{G}(\delta)]$

$p_{52} = \frac{\delta}{\delta+\gamma}$ $\quad$ $p_{56} = \frac{\gamma}{\delta+\gamma}$

$p_{64} = \frac{\beta}{\beta+\delta}[1 - \tilde{H}(\beta+\delta)]$ $\quad$ $p_{65} = \tilde{H}(\beta+\delta)$

$p_{(0)61} = \frac{(\alpha+\beta)\delta}{\delta-\alpha}\left[\frac{1-\tilde{H}(\alpha+\beta)}{(\alpha+\beta)} - \frac{1-\tilde{H}(\beta+\delta)}{(\beta+\delta)}\right]$

$p^{(0)}_{62} = \frac{\delta}{\delta-\alpha}[\tilde{H}(\alpha+\beta) - \tilde{H}(\beta+\delta)]$ ….(25-48)

From the above probabilities the following relation can be verified as;

$p_{01} + p_{02} = 1$ $p_{10} + p_{12} + p_{13} = 1$

$p_{10} + p_{12} + p^{(3)}_{11} + p^{(3)}_{14} = 1$ $p_{20} + p_{24} = 1$

$p_{31} + p_{34} = 1$

$p_{42} + p_{45} + p^{(1)}_{40} + p^{(1)}_{42} + p^{(1)}_{43} = 1$

$p_{42} + p_{45} + p^{(1)}_{40} + p^{(1)}_{42} + p^{(1,3)}_{41} + p^{(1,3)}_{44} = 1$

$p_{52} + p_{56} = 1$ $p_{64} + p_{65} + p^{(0)}_{61} + p^{(0)}_{62} = 1$ ….(49-57)

5.0 Mean Sojourn times:

The mean time taken by the system in a particular state Si before transiting to any other state is known as mean sojourn time and is defined as

$\mu_i = \int_0^\infty P[T>t]\, dt$

where T is the time of stay in state Si by the system.

Therefore

$\mu_0 = \frac{1}{\alpha+\beta}[1 - \tilde{H}(\alpha+\beta)]$ $\mu_1 = \frac{1}{\alpha}[1 - \tilde{G}(\alpha)]$

$\mu_2 = \frac{1}{\alpha+\gamma}$ $\mu_3 = \int_0^\infty t.g(t)dt = m_1$ (say)

$\mu_4 = \frac{1}{\delta}[1 - \tilde{G}(\delta)]$ $\mu_5 = \frac{1}{\delta+\gamma}$

$\mu_6 = \frac{1}{\beta+\delta}[1 - \tilde{H}(\beta+\delta)]$ ….(58-64)

Contribution to Mean Sojourn Time

For the contribution to mean sojourn time in state Si∈E and non-regenerative state occurs, before transiting to Sj∈E, i.e.,

$m_{ij} = -\int_0^\infty t.q_{ij}(t)\, dt = -q'^{*}_{ij}(0)$ ….(65)

Therefore,

$m_{01} = (\alpha+\beta).\int_0^\infty t.e^{-(\alpha+\beta)t}\, \bar{H}(t)dt$

$m_{02} = \int_0^\infty t.e^{-(\alpha+\beta)t}\, h(t)dt$ $m_{10} = q.\int_0^\infty t.e^{-\alpha t}\, g(t)dt$

$m_{12} = p.\int_0^\infty t.e^{-\alpha t}\, g(t)dt$ $m_{13} = \int_0^\infty t.\alpha e^{-\alpha t}\, \bar{G}(t)dt$

$m_{20} = \int_0^\infty t.\gamma e^{-(\alpha+\gamma)t}\, dt$ $m_{24} = \int_0^\infty t.\alpha e^{-(\alpha+\gamma)t}\, dt$

$m_{31} = q.\int_0^\infty t.g(t)dt$ $m_{34} = p.\int_0^\infty t.g(t)dt$

$m_{42} = q.\int_0^\infty t.e^{-\delta t}\, g(t)dt$ $m_{45} = p.\int_0^\infty t.e^{-\delta t}\, g(t)dt$

$m_{52} = \int_0^\infty t.\delta e^{-(\delta+\gamma)t}\, dt = \frac{\delta}{(\delta+\gamma)^2}$

$m_{56} = \int_0^\infty t.\gamma e^{-(\delta+\gamma)t}\, dt = \frac{\gamma}{(\delta+\gamma)^2}$

$m_{64} = \int_0^\infty t.\beta e^{-(\beta+\delta)t}\, \bar{H}(t)dt$ $m_{65} = \int_0^\infty t.e^{-(\beta+\delta)t}\, h(t)dt$

$m^{(3)}_{11} = q.\int_0^\infty t.(1 - e^{-\alpha t})\, g(t)dt$

$m^{(3)}_{14} = p.\ _0\int^\infty t.(1 - e^{-\alpha t})\ g(t)dt$

$m^{(1)}_{40} = q.\ \frac{\delta}{\delta-\alpha}\ [_0\int^\infty t.(e^{-\alpha t} - e^{-\delta t})g(t)dt$

$m^{(1)}_{42} = p.\ \frac{\delta}{\delta-\alpha}\ [_0\int^\infty t.(e^{-\alpha t} - e^{-\delta t})g(t)dt$

$m^{(1,3)}_{41} = q.\ \frac{1}{\delta-\alpha}\ [_0\int^\infty t.\{(\delta-\alpha)-\delta e^{-\alpha t} + \alpha e^{-\delta t}\}g(t)dt]$

$m^{(1,3)}_{44} = p.\ \frac{1}{\delta-\alpha}\ [_0\int^\infty t.\{(\delta-\alpha)-\delta e^{-\alpha t} + \alpha e^{-\delta t}\}g(t)dt]$

$m^{(0)}_{61} = (\alpha+\beta).\ \frac{\delta}{\delta-\alpha}.[_0\int^\infty t.(e^{-\alpha t} - e^{-\delta t})e^{-\beta t}\bar{H}(t)\ dt]$

$m^{(0)}_{62} = \frac{\delta}{\delta-\alpha}.[_0\int^\infty t.(e^{-\alpha t} - e^{-\delta t})e^{-\beta t}h(t)dt]$ ….(66-87)

Hence using (66-87) the following relations can be verified as follows

$m_{01} + m_{02} = \frac{1}{\alpha+\beta}\ [1 - \tilde{H}\ (\alpha+\beta)] = \mu_0$

$m_{10} + m_{12} + m^{(3)}_{11} + m^{(3)}_{14} = {}_0\int^\infty t.g(t)dt\ = m_1$

$m_{20} + m_{24} = \frac{1}{\alpha+\gamma} = \mu_2$

$m_{31} + m_{34} = {}_0\int^\infty t.g(t)dt\ = m_1$

$m_{42} + m_{45} + m^{(1)}_{40} + m^{(1)}_{42} + m^{(1,3)}_{41} + m^{(1,3)}_{44} = {}_0\int^\infty t.g(t)dt = m_1$

$m_{52} + m_{56} = \frac{1}{\delta+\gamma} = \mu_5$

$$m_{64} + m_{65} + m^{(0)}_{61} + m^{(0)}_{62} = \frac{1}{\delta-\alpha}\ [\frac{\beta}{\alpha+\beta}\ \{1 - \tilde{H}(\alpha+\beta)\} - \frac{\alpha}{\alpha+\delta}\ \{1 - \tilde{H}(\alpha+\delta)\}] = m_2$$

….(88-94)

6.0 Reliability and mTsf :

To obtain the reliability of the system we regard all the failed states as absorbing states. If T_i be the time to system failure when it starts operation from state S_i then the reliability of the system is given by

$R_i(t) = Pr[T_i>t]$

Using the probabilistic arguments in view of reliability the following recursive relations can be easily obtained.

$R_0(t) = Z_0(t) + q_{01}(t)©R_1(t) + q_{02}(t)©R_2(t)$

$R_1(t) = Z_1(t) + q_{10}(t)©R_0(t) + q_{12}(t)©R_2(t)$

R2(t) = Z2(t) + q20(t)©R0(t) + q24(t)©R4(t)
R4(t) = Z4(t) + q(1)40(t)©R0(t) + q42(t)©R2(t) + q(1)42(t)©R2(t)
+ q45(t)©R5(t)
R5(t) = Z5(t) + q52(t)©R2(t) + q56(t)©R6(t)
R6(t) = Z6(t) + q(0)61(t)©R1(t) + q(0)62(t)©R2(t) + q64(t)©R4(t)
+ q65(t)©R5(t) ….(95-100)

where

$Z0(t) = e^{-(\alpha+\beta)t}$ $\quad Z1(t) = e^{-\alpha t}\,\overline{G}(t)$

$Z2(t) = e^{-(\alpha+\gamma)t}$ $\quad Z4(t) = \frac{1}{\delta-\alpha}.[\delta e^{-\alpha t} - \alpha e^{-\delta t}]\,\overline{G}(t)$

$Z5(t) = e^{-(\delta+\gamma)t}$ $\quad Z6(t) = \frac{1}{\delta-\alpha}.e^{-\beta t}[\delta e^{-\alpha t} - \alpha e^{-\delta t}]\,\overline{H}(t)$

Taking Laplace transform of the above equations (95-100) and solving for R*0(s) by omitting the argument ‘s’ for brevity, we get

R*0(s) = N1(s)/D1(s) ….(101)

where

N1(s) = q*24q*45q*56q*(0)61(Z*1q*02 – Z*0q*12) - q*45q*56q*64(Z*0
+ Z*1q*01) - q*24(q*42 + q*(1)42)(1 - q*56q*65)(Z*0
+ Z*1q*01) – Z*2q*45q*56q*64(q*01q*12 + q*02)
+ q*24q*45(q*01q*12 + q*02)(Z*5 + Z*6q*56) - q*24q*45(q*52
+ q*56q*(0)62)(Z*0 + Z*1q*01) + Z*4q*24(1 - q*56q*65)
.(q*01 q*12 + q*02) + (1 - q*56q*65)(Z*0 + Z*1q*01+ Z*2q*02
+ Z*2q*01q*12) ….(102)

and

D1(s) = (1 - q*56q*65 – q*45q*56q*64)(1 - q*01q*10 - q*02q*20
- q*01q*12q*20) - q*24(q*42 + q*(1)42)(1 - q*56q*65)
.(1 - q*01q*10) – q*24q*45(q*52 + q*56q*(0)62)(1 - q*01q*10)
- q*24q*45q*56q*(0)61(q*12 + q*10q*02)
- q*24q*(1)40(1 - q*56q*65)(q*02 + q*01q*12)
….(103)

Now,

$$Z^*0(0) = \frac{1}{\alpha+\beta}[1 - \tilde{H}(\alpha+\beta)] = \mu 0$$

$$Z^*1(0) = \frac{1}{\alpha}[1 - \tilde{G}(\alpha)] = \mu 1$$

$$Z^*2(0) = \frac{1}{\alpha+\gamma} = \mu 2$$

$$Z^*4(0) = \frac{1}{\delta-\alpha}.[\frac{\delta}{\alpha}\{1 - \tilde{G}(\alpha)\} - \frac{\alpha}{\delta}\{1 - \tilde{G}(\delta)\}]$$

$$= \frac{1}{\delta-\alpha}.(\delta\mu 1 - \alpha\mu 4) = m3 \text{ (say)}$$

$$Z^*5(0) = \frac{1}{\delta+\gamma} = \mu 5$$

$$Z^*6(0) = \frac{1}{\delta-\alpha}.(\delta\mu 0 - \alpha\mu 6) = m2 \qquad \text{....(104-109)}$$

Therefore,

$$\begin{aligned} N1(0) = &\ p24p45p56p(0)61(\mu 1p02 - \mu 0p12) - p45p56p64(\mu 0 + \mu 1p01) \\ &- p24(p42 + p(1)42)(1 - p56p65)(\mu 0 + \mu 1p01) + p24p45(p01p12 \\ &+ p02)(\mu 5 + m2p56) - \mu 2(p45p56p64(p01p12 + p02) - p24p45(p52 \\ &+ p56p(0)62)(\mu 0 + \mu 1p01) + m3p24(1 - p56p65)(p01\,p12 + p02) \\ &+ (1 - p56p65)(\mu 0 + \mu 1p01 + \mu 2p02 + \mu 2p01p12) \end{aligned} \qquad \text{....(110)}$$

and

$$\begin{aligned} D1(0) = &\ (1 - p56p65 - p45p56p64)(1 - p01p10 - p02p20 - p01p12p20) \\ &- p24(p42 + p(1)42)(1 - p56p65)(1 - p01p10) - p24p45(p52 \\ &+ p56p(0)62)(1 - p01p10) - p24p45p56p(0)61(p12 + p10p02) \\ &- p24p(1)40(1 - p56p65)(p02 + p01p12) \end{aligned} \qquad \text{....(111)}$$

Now the reliability of the system can be obtained by taking inverse Laplace transform of (101) . Also the Mean time to system failure (MTSF) when the system starts operation from state S0 can be obtained as

$$E(T0) = \lim_{s\to 0} R^*0(s) = N1(0)/D1(0) = N1/D1$$

where N1 and D1 are identical as in (110) and (111) respectively.

7.0 Average up time of the system

Let Mi(t) be the probability that starting from state Si the system will be in up position and remains up till epoch t without passing through any regenerative state or returning to itself through one or more non-regenerative points. By probabilistic arguments, we have

$$M0(t) = e^{-(\alpha+\beta)t}\,\overline{H}(t) \qquad M1(t) = e^{-\alpha t}\,\overline{G}(t)$$

$$M2(t) = e^{-(\alpha+\gamma)t}$$

$$M4(t) = \int_0^t \delta e^{-\delta u}\,du.\,\overline{G}(u).\frac{e^{-\alpha(t-u)}\overline{G}(t)}{\overline{G}(u)}$$

$$= \frac{\delta}{\delta-\alpha}.(e^{-\alpha t} - e^{-\delta t})\,\overline{G}(t)$$

$$M6(t) = \int_0^t \delta e^{-(\beta+\delta)u}\,du.\,\overline{H}(u).\frac{e^{-(\alpha+\beta)(t-u)}\overline{H}(t)}{\overline{H}(u)}$$

$$= \frac{\delta}{\delta-\alpha}.e^{-\beta t}(e^{-\alpha t} - e^{-\delta t})\,\overline{H}(t) \qquad \text{....(112-116)}$$

Now, using probabilistic arguments the following recursive relations for point-wise availability Ai(t)'s can be easily obtained

$$A0(t) = M0(t) + q01(t)©A1(t) + q02(t)©A2(t)$$

$$A1(t) = M1(t) + q10(t)©A0(t) + q12(t)©A2(t) + q(3)11(t)©A1(t) + q(3)14(t)©A4(t)$$

$$A2(t) = M2(t) + q20(t)©A0(t) + q24(t)©A4(t)$$

$$A4(t) = M4(t) + q42(t)©A2(t) + q45(t)©A5(t) + q(1)40(t)©A0(t) + q(1)42(t)©A2(t) + q(1,3)41(t)©A1(t) + q(1,3)44(t)©A4(t)$$

$$A5(t) = q52(t)©A2(t) + q56(t)©A6(t)$$

$$A6(t) = M6(t) + q64(t)©A4(t) + q65(t)©A5(t) + q(0)61(t)©A1(t) + q(0)62(t)©A2(t) \qquad \text{....(117-122)}$$

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Taking laplace transform of above equations (117-122) and solving for pointwise availability A*0(s), by omitting the arguments 's' for brevity, one gets

$$A^*0(s) = \frac{N2(s)}{D2(s)} \qquad \ldots(123)$$

where

$$\begin{aligned} N2(s) = {} & [M^*0(1 - q^*(3)11) + M^*1q^*01][(1 - q^*(1,3)44)(1 - q^*56q^*65) \\ & - q^*45q^*56q^*64 - q^*24(q^*42 + q^*(1)42)(1 - q^*56q^*65) \\ & - q^*24q^*45(q^*52 + q^*56q^*(0)62)] - (M^*0q^*12q^*24 + M^*0q^*(3)14 \\ & - M^*1q^*02q^*24 + M^*2q^*02q^*(3)14)[q^*(1,3)44)(1 - q^*56q^*65) \\ & + q^*45q^*56q^*(0)61] + [q^*01\,q^*12 + q^*02(1 - q^*(3)11)] \\ & .[M^*2\{(1 - q^*(1,3)44)(1 - q^*56q^*65) - q^*45q^*56q^*64\} \\ & + q^*24\{M^*4(1 - q^*56q^*65) + M^*6q^*45q^*56\}] \\ & + q^*01\,q^*(3)14\{M^*2\{(q^*42 + q^*(1)42)(1 - q^*56q^*65) + q^*45(q^*52 \\ & + q^*56q^*(0)62) + M^*4(1 - q^*56q^*65) + M^*6q^*45q^*56] \quad \ldots(124) \end{aligned}$$

and

$$\begin{aligned} D2(s) = {} & (1 - q^*(3)11 - q^*01q^*10)[(1 - q^*(1,3)44)(1 - q^*56q^*65) \\ & - q^*45q^*56q^*64 - q^*24(q^*42 + q^*(1)42)(1 - q^*56q^*65) \\ & - q^*24q^*45(q^*52 + q^*56q^*(0)62)] - (q^*12q^*24 + q^*(3)14 \\ & + q^*10q^*02q^*24 - q^*02q^*20q^*(3)14)[q^*(1,3)41(1 - q^*56q^*65) \\ & + q^*45q^*56q^*(0)61\} - [q^*01q^*12 + q^*02(1 - q^*(3)11)] \\ & .[q^*20\{(1 - q^*(1,3)44)(1 - q^*56q^*65) - q^*45q^*56q^*64\} \\ & + q^*24q^*(1)40(1 - q^*56q^*65)] - q^*01\,q^*(3)14\{q^*20\{(q^*42 \\ & + q^*(1)42)(1 - q^*56q^*65) + q^*45(q^*52 + q^*56q^*(0)62)\} \\ & + q^*(1)40(1 - q^*56q^*65)] \end{aligned} \qquad \ldots(125)$$

Also,

$$M^*0(0) = \int e^{-(\alpha+\beta)t}\,\overline{H}(t)dt = \mu 0$$

$$M^*1(0) = \int e^{-\alpha t}\,\overline{G}(t)\,dt = \mu 1$$

$$M^*2(0) = \int e^{-(\alpha+\gamma)t}\,dt = \mu 2$$

$$M^*4(0) = \frac{\delta}{\delta-\alpha}.\int(e^{-\alpha t} - e^{-\delta t})\,\overline{G}(t)\,dt = \frac{\delta}{\delta-\alpha}(\mu 1 - \mu 4) = n1 \text{ (say)}$$

$$M^*6(0) = \frac{\delta}{\delta-\alpha}.\int e^{-\beta t}(e^{-\alpha t} - e^{-\delta t})\,\overline{H}(t)\,dt = \frac{\delta}{\delta-\alpha}(\mu 0 - \mu 6) = n2 \text{ (say)}$$

$$\ldots(126\text{-}130)$$

Also,

$$\begin{aligned} N2(0) = {} & [\mu 0(1 - p(3)11) + \mu 1p01][(1 - p(1,3)44)(1 - p56p65) - p45p56p64 \\ & - p24(p42 + p(1)42)(1 - p56p65) - p24p45(p52 + p56p(0)62)] \\ & - (\mu 0p12p24 + \mu 0p(3)14 - \mu 1p02p24 \\ & + \mu 2p02p(3)14)[p(1,3)44)(1 - p56p65) + p45p56p(0)61] + [p01\,p12 \\ & + p02(1 - p(3)11)][\mu 2\{(1 - p(1,3)44)(1 - p56p65) - p45p56p64\} \\ & + p24\{n1(1 - p56p65) + \mu 6p45p56\}] + p01\,p(3)14\{\mu 2\{(p42 \\ & + p(1)42)(1 - p56p65) + p45(p52 + p56p(0)62) + n1(1 - p56p65) \\ & + n2p45p56] \end{aligned} \qquad \ldots(131)$$

and

$$\begin{aligned} D2(0) = {} & (1 - p(3)11 - p01p10)[(1 - p(1,3)44)(1 - p56p65) - p45p56p64 \\ & - p24(p42 + p(1)42)(1 - p56p65) - p24p45(p52 + p56p(0)62)] \\ & - (p12p24 + p(3)14 + p10p02p24 - p02p20p(3)14) \\ & .[p(1,3)41(1 - p56p65) + p45p56p(0)61\} - [p01p12 \\ & + p02(1 - p(3)11)][p20\{(1 - p(1,3)44)(1 - p56p65) - p45p56p64\} \\ & + p24p(1)40(1 - p56p65)] - p01\,p(3)14\{p20\{(p42 + p(1)42) \end{aligned}$$

.(1 - p56p65) + p45(p52 + p56p(0)62)} + p(1)40(1 - p56p65)]
= 0

Now, for obtaining D'2(0) the coefficients of mij's in D'2(0) are,

m01 → (p10p24 – p20p(3)14)[p(1,3)44)(1 - p56p65) + p45p56p(0)61 + (1 – p(3)11)[p20{(1 - p(1,3)44)(1 - p56p65) - p45p56p64} + p24p(1)40(1 - p56p65)] = L0 (say)

m02 → (p10p24 – p20p(3)14)[p(1,3)44)(1 - p56p65) + p45p56p(0)61 + (1 – p(3)11)[p20{(1 - p(1,3)44)(1 - p56p65) - p45p56p64} + p24p(1)40(1 - p56p65)] = L0

m10 → p24[p(1,3)41(1 - p56p65) + p45p56p(0)61 + p01[p20{(1 - p(1,3)44)[(1 - p56p65) - p45p56p64} + p24p(1)40(1 - p56p65)] = L1 (say)

m12 → p24[p(1,3)41(1 - p56p65) + p45p56p(0)61 + p01[p20{(1 - p(1,3)44)[(1 - p56p65) - p45p56p64} + p24p(1)40(1 - p56p65)] = L1

m(3)11 → p24[p(1,3)41(1 - p56p65) + p45p56p(0)61 + p01[p20{(1 - p(1,3)44)[(1 - p56p65) - p45p56p64} + p24p(1)40(1 - p56p65)] = L1

m(3)14 → p24[p(1,3)41(1 - p56p65) + p45p56p(0)61 + p01[p20{(1 - p(1,3)44)[(1 - p56p65) - p45p56p64} + p24p(1)40(1 - p56p65)] = L1

m20 → p02p(3)14[p(1,3)41(1 - p56p65) + p45p56p(0)61] + [p01p12 + p02(1 - p(3)11)][(1 - p(1,3)44)(1 - p56p65) - p45p56p64} + p01p(3)14[(p(1)42 + p42)(1 - p56p65) + p45(p52 + p56p(0)62] = L2 (say)

m24 → p02p(3)14[p(1,3)41(1 - p56p65) + p45p56p(0)61] + [p01p12 + p02(1 - p(3)11)][(1 - p(1,3)44)(1 - p56p65) - p45p56p64} + p01p(3)14[(p(1)42 + p42)(1 - p56p65) + p45(p52 + p56p(0)62] = L2

m42 → (1 - p56p65).{1 – p12p24 + p(3)14 + p14p02p24 – p02p20p(3)14 = (1 - p56p65).L4 (say)

m45 → (1 - p56p65).L4

m(1)40 → (1 - p56p65).L4

m(1)42 → (1 - p56p65).L4

m(1,3)41 → (1 - p56p65).L4

m(1,3)44 → (1 - p56p65).L4

m52 → p24p45(1 – p(3)11 – p01p10) + p02p(3)14p45 = L5 (say)

m56 → p24p45(1 – p(3)11 – p01p10) + p02p(3)14p45 = L5

m64 → p45p56L4

m65 → p45p56L4

m(0)61 → p45p56L4

m62 → p45p56L4

Thus,

D2 = D'2(0) = (m01 + m02)L0 + (m10 + m12 + m(3)11 + m(3)14)L1
+ (m20+m24)L2 + (m42 + m45 + m(1)40 + m(1)42 + m(1,3)41
+ m(1,3)44)(1 – p56p65).L4 + (m52 + m56).L5 + (m64 + m65
+ m(0)61 + m(0)62)p45p56.L4
= μ0L0 + m1L1 + μ2L2 + m1(1 – p56p65)L4 + μ5L5 + m2p45p56.L4
= μ0L0 + m1[L1 + L4(1 – p56p65)] + μ2L2 + μ5L5 + m2p45p56L4 ….(132)

Therefore, by L. Hospital rule

A0 = N2(0)/D'2(0) = N2/D2

where N2 and D2 are same as in equations (131) and (132) respectively.

8.0 Conclusion:

Mathematical results of reliability analysis of a production unit obtained in this paper are useful in improving reliability of industrial production units. The model discussed in the paper is useful for managers for running a production unit effectively.

9.0 References

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